Name: NETTY

Math 10170, Final Exam
May 7, 2015

Instructor: ______________________

- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.
- You may use your **calculator** and a **formula sheet** during this exam.
- **Turn off and put away all cell-phones** and similar electronic devices.
- **Head phones** are not allowed.
- **Put away all notes except your formula sheet** where they cannot be viewed.
- The exam lasts for TWO HOURS.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 17 pages of the test.
- Hand in the entire exam.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)	15.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)						
13.	(a)	(b)	(c)	(d)	(e)						
14.	(a)	(b)	(c)	(d)	(e)						

Multiple Choice

1.(6 pts.) There are 12 teams competing in a round robin tournament where every team plays every other team exactly once. How many games must be played?

(a) 42 (b) 66 (c) 12 (d) 60 (e) 78

The number of games necessary in a round robin tournament with n teams is $\frac{n(n-1)}{2}$

When $n=12$, # games $= \frac{12(11)}{2} = 66$

2.(6 pts.) You are considering choosing one of the 5 quarterbacks listed below for your fantasy football team. The rankings shown below are the rankings from 4 of the best fantasy football websites. WSA, WSB, WSC and WSD.

	WSA	**WSB**	**WSC**	**WSD**	AVE.
P. Manning	1	2	3	1	$(1+2+3+1)/4 = 7/4$
T. Brady	2	1	1	4	$8/4$
A. Luck	3	3	2	2	$10/4$
A. Rogers	4	4	4	3	$15/4$
T. Romo	5	5	5	5	$20/4$

Using the Borda Method to amalgamate the rankings, which quarterback should I choose?

~~(a)~~ P. Manning (b) A. Rogers (c) A. Luck

(d) T. Romo (e) T. Brady

The person WITH THE (NUMERICALLY) LOWEST AVERAGE RANKING IS P. MANNING.

3.(6 pts.) Consider the following matrices:

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 4 \\ -1 & -2 & -4 \end{pmatrix} \qquad C = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Which of the following matrices is equal to $(A+B)C$?

(a) $\begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix}$. (b) $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ (c) $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$. (d) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (e) $\begin{pmatrix} 12 \\ -3 \end{pmatrix}$

$$(A+B)\cdot C = \begin{pmatrix} 2 & 2 & 6 \\ -2 & -1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2-2+12 \\ -2+1-2 \end{pmatrix} = \begin{pmatrix} 12 \\ -3 \end{pmatrix}$$

4.(6 pts.) Five athletes are contenders for the "best athlete" award at a gymnastics competition. All five athletes have competed in 7 events at the competition. The judges wish to give the prize to the Condorcet winner for these seven events, if there is one (otherwise they use a completion method). The results of the seven events are shown below (the events are labelled $E1, E2, \ldots E7$) :

Events

	E1	**E2**	**E3**	**E4**	**E5**	**E6**	**E7**
Adam Armstrong	2	2	3	4	1	5	1
Barney Begley	1	5	1	2	3	4	2
Ciaran Callan	3	1	2	1	2	2	5
Dennis Dineen	4	4	4	5	4	3	4
Eoin Ennis	5	3	4	3	5	1	3

The Condorcet winner is:

(a) Dennis Dineen (b) Ciaran Callan (c) Adam Armstrong

(d) Barney Begley (e) Eoin Ennis

PAIRWISE COMPARISONS:

AA v BB → BB
3 v 4

BB v CC → CC
3 v 4

CC v DD → CC
6 v 1

CC v EE → CC
5 v 2

CC v AA → CC
4 v 3

CC is THE CONDORCET WINNER since CC beats ALL OTHER Player IN A HEAD TO HEAD COMPARISON.

5.(6 pts.) Consider the following system of linear equations.

$$\begin{aligned} x - y + z + 2w &= 7 \\ x + z - w &= -1 \\ -x + 2y - 3z + 5w &= 2 \end{aligned}$$

Which of the following gives the corresponding matrix equation?

(a) $$\begin{pmatrix} 1 & -1 & 1 & 2 \\ 1 & 0 & 1 & -1 \\ -1 & 2 & -3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(b) $$\begin{pmatrix} 1 & -1 & 1 & 2 \\ 1 & 0 & 1 & -1 \\ -1 & 2 & -3 & 5 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \\ 2 \\ 0 \end{pmatrix}$$

~~(c)~~ $$\begin{pmatrix} 1 & -1 & 1 & 2 \\ 1 & 0 & 1 & -1 \\ -1 & 2 & -3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \\ 2 \end{pmatrix}$$

when we multiply The MATRICES ON THE LEFT AND EQUATE THE RESULT WITH THE MATRIX ON THE RIGHT, we get the above set of equations.

(d) $$\begin{pmatrix} 1 & -1 & 1 & 2 \\ 1 & 1 & -1 & 0 \\ -1 & 2 & -3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \\ 2 \end{pmatrix}$$

(e) None of the above

6.(6 pts.) Consider the following system of linear equations.

$$\begin{aligned} x - y + z + 2w &= 6 \\ x + z - w &= 2 \\ -x + 2y - 3z + 5w &= -4 \end{aligned}$$

Which of the following gives a solution to this system of equations?

(a) $x = 1,\ y = -1,\ z = 2,\ w = -1$ (b) $x = -1,\ y = -1,\ z = 2,\ w = 1$

(c) $x = 0,\ y = -1,\ z = -2,\ w = 1$ (d) $x = 1,\ y = -1,\ z = 2,\ w = 1$

(e) $x = 2,\ y = -1,\ z = 1,\ w = 1$

EQUATIONS
$x - y + z + 2w = 6$
$x + z - w = 2$
$-x + 2y - 3z + 5w = -4$

(a) $1 - (-1) + 2 + 2(-1) = 6$
NOT A SOLUTION

(b) $(-1) - (-1) + 2 + 2 = 6$
NOT A SOLUTION

(c) $0 - (-1) + (-2) + 2(1) = 6$
NOT A SOLUTION

(d) $1 - (-1) + 2 + 2 = 6$ ✓
$1 + 2 - 1 = 2$ ✓
$-1 + 2(-1) - 3(2) + 5(1) = -4$ ✓

(d) is a solution to all 3 equations

7.(6 pts.) If I roll a six sided die twice and observe the numbers on the uppermost faces, the sample space is given by:

B

$A \cap B$

A

$$\begin{array}{llllll} \{(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6)\} \end{array}$$

Let A be the event that I observe a 3 on the first roll and let B be the event that the product of the numbers is less than or equal to 6.

Which of the following gives $P(A|B)$. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{2/36}{14/36} = \dfrac{1}{7}$

(a) $\frac{1}{18}$ (b) $\frac{1}{3}$ (c) $\frac{1}{10}$ (d) $\frac{1}{9}$ (e) $\frac{1}{7}$

8.(6 pts.) Let

$$A = \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix}.$$

Which of the following is A^{-1}?

(a) $A^{-1} = \begin{pmatrix} -7 & 3 \\ 5 & -1 \end{pmatrix}$

(b) $A^{-1} = \begin{pmatrix} \frac{7}{8} & -\frac{3}{8} \\ -\frac{5}{8} & \frac{1}{8} \end{pmatrix}$

(c) $A^{-1} = \begin{pmatrix} -\frac{1}{8} & -\frac{3}{8} \\ -\frac{5}{8} & -\frac{7}{8} \end{pmatrix}$

(d) $A^{-1} = \begin{pmatrix} 7 & -3 \\ -5 & 1 \end{pmatrix}$

~~(e)~~ $A^{-1} = \begin{pmatrix} -\frac{7}{8} & \frac{3}{8} \\ \frac{5}{8} & -\frac{1}{8} \end{pmatrix}$

DET A = 7 − 15 = − 8

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

THEN $A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Here $A^{-1} = -\frac{1}{8}\begin{pmatrix} 7 & -3 \\ -5 & 1 \end{pmatrix} = \begin{pmatrix} -7/8 & 3/8 \\ 5/8 & -1/8 \end{pmatrix}$

9.(6 pts.) In an experiment, a basketball player with a 90% chance of making a basket (B) on every shot shoots until she misses a basket (M) . The first time she misses a basket, she stops and records the outcome as a sequence of B's and M's, where B denotes a basket made and M denotes a basket missed. What is the probability that the outcome of this experiment is

$$BBBBBM$$

(a) $1-(0.5)^6$ ~~(b)~~ $(0.9)^5(0.1)$ (c) 0

(d) $(0.5)^6$ (e) $(0.1)^5(0.9)$

P(BBBBBM)

$= (.9)^5(0.1)$

10.(6 pts.) The following table shows the results from the first three weeks of a round robin soccer.

	Kirawee	Sutherland	Gymea	Sylvania	Woronora	Miranda
Kirawee			5-3	2-1	2-3	
Sutherland			5-1		4-1	1-2
Gymea	3-5	1-5		1-4		2-3
Sylvania	1-2		4-1			2-5
Woronora	3-2	1-4				
Miranda		2-1	3-2	5-2		

Which of the following matrix equations **must be solved** in order to find the Massey Ratings (keeping the same ordering of the teams as above)?

(a)
$$\begin{pmatrix} 3 & 0 & -1 & -1 & -1 & 0 \\ 0 & 3 & -1 & 0 & -1 & -1 \\ -1 & -1 & 4 & -1 & 0 & -1 \\ -1 & 0 & -1 & 3 & 0 & -1 \\ -1 & -1 & 0 & 0 & 2 & 0 \\ 0 & -1 & -1 & -1 & 0 & 3 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ -10 \\ -1 \\ -2 \\ 5 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 5 & 0 & -1 & -1 & -1 & 0 \\ 0 & 5 & -1 & 0 & -1 & -1 \\ -1 & -1 & 6 & -1 & 0 & -1 \\ -1 & 0 & -1 & 5 & 0 & -1 \\ -1 & -1 & 0 & 0 & 4 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 3/2 \\ 3/2 \\ 0 \\ 1/2 \\ 0 \\ 0 \end{pmatrix}$$

#games Played by Team 1

Point Differential for Team 1

(~~c~~)
$$\begin{pmatrix} 3 & 0 & -1 & -1 & -1 & 0 \\ 0 & 3 & -1 & 0 & -1 & -1 \\ -1 & -1 & 4 & -1 & 0 & -1 \\ -1 & 0 & -1 & 3 & 0 & -1 \\ -1 & -1 & 0 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ -10 \\ -1 \\ -2 \\ 0 \end{pmatrix}$$

#games between Team i and Team j

bottom Row Replaced by 1's and a 0 on Right

(d)
$$\begin{pmatrix} 5 & 0 & -1 & -1 & -1 & 0 \\ 0 & 5 & -1 & 0 & -1 & -1 \\ -1 & -1 & 6 & -1 & 0 & -1 \\ -1 & 0 & -1 & 5 & 0 & -1 \\ -1 & -1 & 0 & 0 & 4 & 0 \\ 0 & -1 & -1 & -1 & 0 & 5 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 3/2 \\ 3/2 \\ 0 \\ 1/2 \\ 0 \\ 3/2 \end{pmatrix}$$

Colley

(e) None of the above

11.(6 pts.) If a basketball player with a 70% chance of making a basket on every shot, takes 200 consecutive shots, what is the expected length of the longest run of baskets that would appear in the data?

~~(a)~~ approx. 11 (b) approx. 5 (c) approx. 3

(d) approx. 7 (e) approx. 20

$K = 200$

$p = 0.7$

EXPECTED LENGTH OF LONGEST RUN

$$\approx -\frac{\ln((1-p)K)}{\ln(p)}$$

$$= -\frac{\ln(0.3(200))}{\ln(0.7)} \approx 11.479 \approx 11$$

12.(6 pts.) An experiment consists of flipping a coin 3 times. Which of the following shows the distribution of the random variable X associated to this experiment, where X denotes the number of heads in the resulting sequence.

(a)

X	P(X)
0	2/8
1	2/8
2	2/8
3	2/8

(b)

X	P(X)
1	3/8
2	3/8
3	2/8

(c)

X	P(X)
1	3/8
2	2/8
3	3/8

~~(d)~~

X	P(X)
0	1/8
1	3/8
2	3/8
3	1/8

(e)

X	P(X)
0	1/8
1	1/2
2	1/2
3	1/8

OUTCOME	VALUE OF X
HHH	3
HHT	2
HTH	2
HTT	1
THH	2
THT	1
TTH	1
TTT	0

X	P(X)
0	1/8
1	3/8
2	3/8
3	1/8

13.(6 pts.) An experiment consists of rolling a pair of fair six sided dice. The distribution of the random variable X is shown below where X denotes the larger number minus the smaller number on the uppermost faces.

X	P(X)	$XP(X)$
0	6/36	0
1	10/36	10/36
2	8/36	16/36
3	6/36	18/36
4	4/36	16/36
5	2/36	10/36

$E(X) = 70/36 = 1.94$

(a) $E(X) \approx 3$ (b) $E(X) \approx 6$ (c) $E(X) \approx 3.21$

(d) $E(X) \approx 1.94$ (e) $E(X) \approx 2.42$

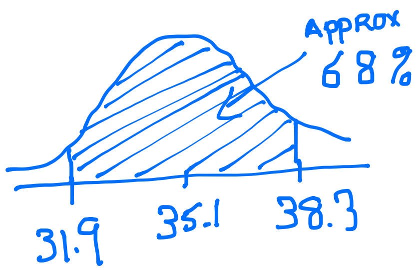
14.(6 pts.) Statistics collected over a five year period for performance of wide receivers in the vertical jump at the NFL combine showed an average of 35.1 inches and a standard deviation of 3.2. Assuming that the distribution of the observations was mound shaped (roughly normally distributed), roughly what percentage of the wide receivers had a vertical jump height between 31.9 inches and 38.3 inches.

(a) 68% (b) 50% (c) 80%

(d) 99.7% (e) 95%

$\mu = 35.1$

$\sigma = 3.2$



If $X = 31.9$

$$\frac{X-\mu}{\sigma} = \frac{31.9-35.1}{3.2} = -1$$

31.9 is 1 st. Dev. below mean

If $X = 38.3$

$$\frac{X-\mu}{\sigma} = \frac{38.3-35.1}{3.2} = 1$$

THEREFORE
38.3 is 1 ST. DEV. Above THE MEAN

15.(6 pts.) Julio is making a weekly workout plan for the semester. He has downloaded three workout videos from the internet.

- Workout A takes 15 minutes, burns 150 calories and involves 2 minutes of anaerobic training.
- Workout B takes 20 minutes, burns 250 calories and involves 5 minutes of anaerobic training.
- Workout C takes 15 minutes, burns 200 calories and involves 6 minutes of anaerobic training.

Julio wants to work out for

- a total of 180 minutes each week,
- he wants to burn 2250 calories per week and
- he wants to get 55 minutes of anaerobic training each week.

Let

- x denote the number of times Julio will complete Workout A each week,
- let y denote the number of times Julio will complete Workout B each week and
- let z denote the number of times Julio will complete Workout C each week.

Which system of equations must he solve:

(a)
$$\begin{aligned} 15x + 150y + 2z &= 180 \\ 20x + 250y + 5z &= 220 \\ 15x + 200y + 6z &= 55 \end{aligned}$$

(b)
$$\begin{aligned} 20x + 15y + 20z &= 180 \\ 150x + 200y + 150z &= 220 \\ 5x + 5y + 2z &= 55 \end{aligned}$$

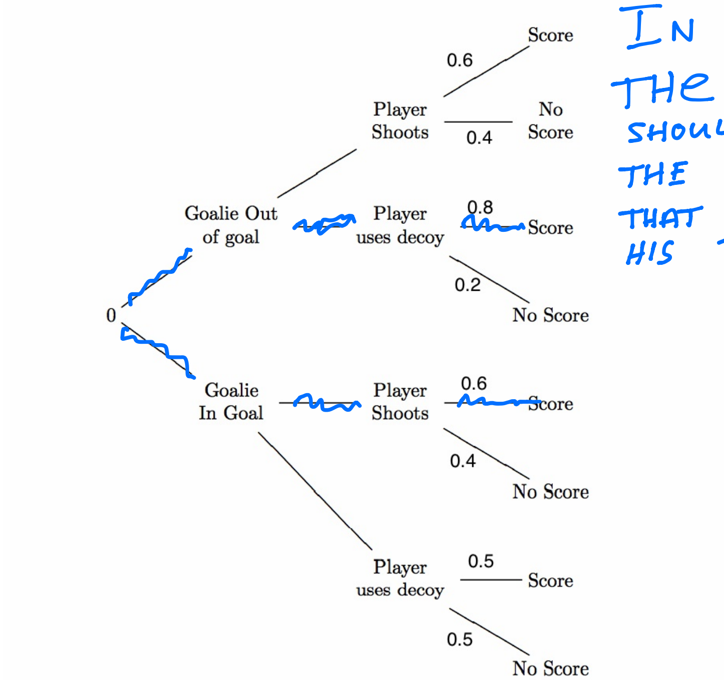
(c)
$$\begin{aligned} 30x + 20y + 0.1z &= 100 \\ 10x + 40y + 0z &= 100 \\ 0x + 0y + 50z &= 100 \end{aligned}$$

(d)
$$\begin{aligned} 15x + 20y + 15z &= 180 \\ 150x + 200y + 200z &= \cancel{220}\ 2250 \\ 2x + 5y + 6z &= 55 \end{aligned}$$

(e)
$$\begin{aligned} 15x + 200y + 10z &= 180 \\ 150x + 20y + 200z &= 220 \\ 2x + 50y + 60z &= 55 \end{aligned}$$

	x A	y B	z C	TOTAL
TIME	15	20	15	180
CAL	150	200	200	2250
ANAEROBIC	2	5	6	55

16.(6 pts.) A hockey player has made estimates of the probability of scoring a goal against a particular goalie with the strategies of using a decoy or shooting depending on whether the goalie is in the goal or out of the goal. His results are shown in the tree diagram below:



IN EACH CASE THE PLAYER SHOULD CHOOSE THE STRATEGY THAT MAXIMIZES HIS PROB OF SCORING

Which of the following describes the player's best strategy:

(a) If the goalie is out of the goal, the player should shoot 50% of the time and use a decoy 50% of the time
and if the goalie is in the goal, the player should always use a decoy.

(b) The player should always shoot

(c) If the goalie is in the goal, the player should shoot and
if the goalie is out of the goal, the player should use a decoy

(d) The player should always use a decoy

(e) If the goalie is in the goal, the player should use a decoy and
if the goalie is out of the goal, the player should shoot

17.(6 pts.) A ball is thrown directly upwards at a speed of 5 meters per second. What is the maximum height above the starting point reached by the ball? (We assume that the only force acting on the ball after it is thrown is that of gravity giving a negative vertical acceleration of 9.8 m/s^2.)

(a) approx. 2.55 meters

(b) approx. 3.12 meters

(c) approx. 0.51 meters

(d) approx. 10.18 meters

(e) approx. 1.27 meters

NOT ON EXAM.

18.(6 pts.) Find equilibrium points of the following payoff matrix for players R and C:

	C1	C2	C3	C4	MAX C
R1	(1,4)	(-1, 3)	(1, 7)	(3, 6)	7
R2	(-2,10)	(6, 7)	(-1, 3)	(2, -1)	10
R3	(-2,7)	(4, 8)	(-1, 7)	(5, 10)	10
R4	(3,6)	(7, 2)	(1, 3)	(-1, 2)	6
MAX R	3	7	1	5	

(a) There is a unique equilibrium point at $R4C1$

(b) There is a unique equilibrium point at $R1C3$

(c) There are exactly two equilibrium points at $R4C1$ and $R3C4$ only

(d) There are exactly 3 equilibrium points at R4C1, R1C3, R3C4.

(e) There are no equilibrium points

19.(6 pts.) Two kickboxers, Rich and Chris are facing each other. When Chris attacks, he uses one of three techniques; a jab and reverse punch combination (J&R), a round punch to the head (R) or a front kick (FK). Rich has three strategies for defending; move to avoid the technique and counterattack (C), block the attack and counter (B) or move in very quickly and punch as Chris attacks (hopefully) hitting Chris before Chris' attack lands (A). The payoff matrix for Rich shown below gives the probability that Rich will score a point for each of the nine possible scenarios:

	J&R	R	FK
C	0.9	0.6	0.8
B	0.3	0.8	0.5
~~A~~	~~0.2~~	~~0.5~~	~~0.3~~

B DOMINATES A
THERE ARE NO DOMINATED STRATEGIES IN THE REDUCED MATRIX.

Find the reduced payoff matrix

(a)

	J&R	R	FK
C	0.9	0.6	0.8
B	0.3	0.8	0.5

(b)

	J&R	R	FK
C	0.9	0.6	0.8
A	0.2	0.5	0.3

(c)

	J&R	FK
C	0.9	0.8
B	0.3	0.5

(d)

	J&R
C	0.9

(e)

	J&R	R
C	0.9	0.6
B	0.3	0.8

20.(6 pts.) For the two High school football teams the Ravens and the Crows, the coach of the Ravens estimates the expected number of yards gained for the Ravens in each possible situation, depending on whether the Ravens run the ball or pass it and whether the Crows execute a pass defense or a run defense.

- If the Ravens run the ball (R) and the Crows choose a run defense (RD), the Ravens lose 2 yards on average. -2 (R, RD)
- If the Ravens run the ball (R) and the Crows choose a pass defense (PD), the ravens gain 5 yards on average. +5 (R, PD)
- If the Ravens pass the ball (P) and the Crows choose a pass defense (PD), the ravens gain 0 yards on average. 0 (P, PD)
- If the Ravens pass the ball (P) and the Crows choose a run defense (RD), the ravens gain 7 yards on average. 7 (P, RD)

Which of the following shows the correct payoff matrix for The Ravens for this zero sum game?

(a)

		Carl	
		RD	PD
Robert	R	7	0
	P	-2	5

X (b)

		~~**Carl**~~ CROWS	
		RD	PD
~~**Robert**~~ RAVENS	R	-2	5
	P	7	0

(c)

		Carl	
		RD	PD
Robert	R	5	-2
	P	0	7

(d)

		Carl	
		RD	PD
Robert	R	-2	7
	P	0	5

(e)

		Carl	
		RD	PD
Robert	R	2	5
	P	7	0

21.(6 pts.) In a simplified penalty shoot out in soccer, a goalie has two options; either dive right (DR) or dive left (DL). The penalty taker also has two options, kick left (KL) or kick right (KR). Assume the probability of a goal for the penalty taker is shown in the payoff matrix below, where the row player is the penalty taker.

		Goalie DL	DR
P. Taker R	KL	0.5	0.7
	KR	0.8	0.6

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Which of the following gives the penalty taker's optimal strategy for this game?

(a) $\left(\frac{1}{4}, \frac{3}{4}\right)$ (b) $\left(\frac{3}{4}, \frac{1}{4}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{4}{5}, \frac{1}{5}\right)$ (e) $\left(\frac{1}{5}, \frac{4}{5}\right)$

R's OPTIMAL STRATEGY
$(p, 1-p)$
$\left(\frac{1}{2}, \frac{1}{2}\right) \leftarrow p = \frac{d-c}{(a+d)-(b+c)} = \frac{0.6-0.8}{(0.5+0.6)-(0.8+0.7)} = \frac{-0.2}{-0.4} = \frac{1}{2}$

22.(6 pts.) In a simplified penalty shoot out in soccer, a goalie has two options; either dive right (DR) or dive left (DL). The penalty taker also has two options, kick left (KL) or kick right (KR). Assume the probability of a goal for the penalty taker is shown in the payoff matrix below, where the row player is the penalty taker.

		Goalie C DL	DR
P. Taker	KL	0.5	0.7
	KR	0.8	0.6

Which of the following gives the goalie's optimal strategy for this game?

(a) $\begin{pmatrix} \frac{1}{4} \\ \frac{3}{4} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$ (c) $\begin{pmatrix} \frac{1}{5} \\ \frac{4}{5} \end{pmatrix}$ (d) $\begin{pmatrix} \frac{3}{4} \\ \frac{1}{4} \end{pmatrix}$ (e) $\begin{pmatrix} \frac{4}{5} \\ \frac{1}{5} \end{pmatrix}$

C's OPTIMAL STRATEGY
$\begin{pmatrix} q \\ 1-q \end{pmatrix}$
$q = \frac{d-b}{(a+d)-(b+c)} = \frac{0.6-0.7}{-0.4} = \frac{-0.1}{-0.4} = \frac{1}{4}$

$= \begin{pmatrix} \frac{1}{4} \\ \frac{3}{4} \end{pmatrix}$

23.(6 pts.) In a simplified penalty shoot out in soccer, a goalie has two options; either dive right (DR) or dive left (DL). The penalty taker also has two options, kick left (KL) or kick right (KR). Assume the probability of a goal for the penalty taker is shown in the payoff matrix below, where the row player is the penalty taker.

		Goalie	
		DL	DR
P. Taker	KL	0.5	0.7
	KR	0.8	0.6

Which of the following gives the **expected payoff for the Goalie** for this game?

$= 1 - \nu$

(a) 0.25 (b) 0.5 (c) ~~0.65~~ 0.35 (d) 0.26 (e) 0.75

$$\nu = \frac{ad - bc}{(a+d) - (b+c)} = \frac{(0.5)(0.6) - (0.8)(0.7)}{-0.4} = \frac{-0.26}{-0.4} = 0.65$$

$$\boxed{1 - \nu = 0.35}$$

24.(6 pts.) A golfer hits the ball from the tee on level ground at an initial speed of 42 meters per second and an initial angle of 50^0 above the horizontal. (We assume that the only force acting on the ball throughout its flight is that of gravity giving a negative vertical acceleration of 9.8 m/s^2.)
What is the maximum height reached by the ball?

(a) approx. 26.99 m. (b) approx. 32.17 m. (c) approx. 52.8 m.

(d) approx. 105.5 m. (e) approx. 177.1 m.

NOT ON EXAM

25.(6 pts.) A golfer hits the ball from the tee on level ground at an initial speed of 42 meters per second and an initial angle of 50^0 above the horizontal. (We assume that the only force acting on the ball throughout its flight is that of gravity giving a negative vertical acceleration of 9.8 m/s^2.)
What is horizontal distance covered by the ball when it first hits the ground?

(a) approx. 105.5 m. (b) approx. 177.1 m. (c) approx. 26.99 m.

(d) approx. 32.17 m. (e) approx. 52.8 m.

NOT ON EXAM